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# Film condensation in presence of non-condensable gases over horizontal tubes with progressively increasing radius of curvature in the direction of gravity

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#### Abstract

A theoretical study is executed to investigate the simultaneous influences of reduction in heat transfer rate on account of condensation in presence of non-condensables and its augmentation by the geometry of an horizontal tube surface with increasing radius of curvature in the direction of gravity. The tube surface profile considered for the present work is an equiangular spiral described in a polar form as:  $R = ae^{m\theta}$  (a and m being parametric constants). It is observed that a very small bulk concentration (even less than 1%) of the non-condensable gas reduces considerably the heat transfer coefficient. However, there is an enhancement in heat transfer coefficient for condensation over a polar tube surface, as compared to that over a circular tube surface. This enhancement in heat transfer coefficient, with an increase in the value of  $m$  (a surface profile parameter), in presence of non-condensables is more than the corresponding proportional enhancement in the same in absence of any non-condensable species.

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# 1. Introduction

The phenomenon of condensation on outer and inner surface of horizontal tubes is largely associated with various fields of engineering such as thermal power plant, refrigeration, air conditioning and many chemical processing industries. Most of the condensers for commercial purposes use circular tubes carrying cooling water through them. The two important aspects that play prominent role in the condensation heat transfer rate are tube geometry and presence of non-condensable gases in the vapour.

A pioneering work in the field of condensation heat transfer is due Nusselt [\[1\],](#page-6-0) who predicted the heat transfer coefficient in film condensation over horizontal circular tubes, from a simplified theoretical analysis. Several works on [\[2–8\]](#page-6-0) laminar film condensation over non-circular tubes have been reported subsequently, following Nusselt's basic theory. These studies have basically originated out of a motivation to investigate the role of tube surface geometry in enhancing the rate of heat transfer during condensation. In a very recent work, Dutta et al. [\[9\]](#page-6-0) established the fact that a considerable augmentation in heat transfer coefficient takes place for condensation over non-circular tubes with progressively increasing radius of curvature in the direction of gravity, as compared to that over a conventional circular tube. This has been attributed to a combined effect of gravity force component and surface tension driven pressure gradient in the flow of liquid film along the solid surface.

In many practical situations, condensable vapours contain certain non-condensable gases. It is well known that the non-condensable gases drastically reduce the condensation heat transfer rate due to their presence even by a very minute amount. In such situations, concentration and

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## Nomenclature

- a parametric constant of polar curve  $R_p = ae^{m\theta}$
- C mass fraction of vapour
- D mass diffusivity
- $c_p$  specific heat of condensate at mean film temperature
- g acceleration due to gravity
- $h_{\text{fe}}$  latent heat due to condensation
- $Ja$  Jacob number,  $\frac{c_p(T_{si}-T_w)}{h_{fg}}$
- k thermal conductivity of condensate at mean film temperature
- L length of semi-perimeter of any polar curve
- m parametric constant of polar curve  $R_p = ae^{m\theta}$
- $\dot{m}_c$  mass flux of condensate
- $N_{\sigma}$  non-dimensional surface tension force,  $\frac{\sigma}{(\rho \rho_{\rm v}) g R^2}$
- $P_{\sigma}$  pressure due to surface tension<br>  $P$  perimeter of the tube surface
- perimeter of the tube surface
- Q condensate volume flow rate per unit width of the surface R radius of an equivalent circle having same
- surface area to that of the polar curve
- $R<sub>p</sub>$  radius vector of polar curve
- $\overrightarrow{R_{c}}$  radius of curvature r radial coordinate
- Ra Rayleigh number,  $\frac{\rho(\rho \rho_v)g(2R)^3c_p}{\mu k}$
- 

ical analysis, the rate of condensation heat transfer rate is predicted, in order to quantitatively assess the role of the tube geometry in arresting the reduction of rate of heat transfer on account of presence of non-condensables in the system.

### 2. Mathematical modeling

A central theme of the present work is to investigate the effects of non-condensables on condensation heat transfer on a tube surface, which is having a profile such that its radius of curvature increases in the direction of gravity. The geometric profile chosen for this purpose is given by the polar equation of an equiangular spiral as:  $R_p = ae^{m\theta}$ , which is generated symmetrically about its vertical axis. However, the entire symmetrical half of the polar curve ([Fig. 1\(](#page-2-0)a)) cannot be taken to model the tube surface, since there exists an upward sloping portion in which the condensate cannot flow with the aid of gravity. If the polar angle  $\theta$ is measured with respect to the central vertical axis, with a positive sense in the clockwise direction, the above zone of upward facing slope is confined within a region given by:  $0 \le \theta \le \tan^{-1} m$  (the portion AB shown in [Fig. 1](#page-2-0)(b)). It can be noted here that  $m = 0$  corresponds to the special case of a circular cylinder, which does not contain any upward sloping portion at all. This leads to the consideration of a surface formed by the segment of the polar profile  $R_p = ae^{m\theta}$  described on a vertical chord BE as shown in

- $S_p$  semi-perimeter of the tube surface<br>  $T$  temperature
- temperature
- $T_{\rm w}$  wall temperatures
- $V_{\rm{si}}$  condensate velocity along tangential direction of the tube surface

Greek symbols

- $\alpha$  thermal diffusivity
- $\beta$  angle between tangent and direction due to gravity at any point on tube surface
- $\delta$  liquid film thickness
- $\theta$  polar angle
- $\rho$  density of condensate at mean film temperature
- $\rho_{\rm v}$  density of vapour
- $\mu$  dynamic viscosity of condensate at mean film temperature
- $\sigma$  surface tension coefficient

#### Subscripts

- $x_1$  tangential direction along the tube surface
- $y_1$  normal direction at any point to the tube surface
- v vapour phase
- $\infty$  at free stream

temperature gradients are set up with combined heat and mass transport processes in the vapour–gas mixture. A number of analytical and semi-analytical investigations [\[10–13\]](#page-6-0) has been executed in the literature on this issue, referring mostly to the condensation on vertical flat surfaces. These studies have pointed out that a substantial reduction in condensation heat transfer coefficient due to presence of non-condensable species is likely to occur. In some of these works, a simplification has been made by locally replacing the saturation temperature of the vapour in Nusselt's equation with interface temperature of the mixture, as solved from the momentum, heat and species conservation equations. However, studies on condensation over a curved surface in presence of non-condensables have received only a little attention in the literature.

Major aim of the present work is to investigate the simultaneous influences of reduction of heat transfer rate due to presence of non-condensables and its augmentation by the geometry of the tube surface with increasing radius of curvature in the direction of gravity. This is mathematically achieved by obtaining the evolution of film thickness in an iterative manner, from a coupled solution of momentum, heat and species conservation equation, in conjunction with matching interfacial constraints and boundary conditions. This is in sharp contrast with some of the earlier approaches, in which validity of a classical form of Nusselt's equation has been implicitly presumed, even in the presence of non-condensables. Based on the mathemat-

<span id="page-2-0"></span>

Fig. 1. An equiangular spiral curve generated on vertical axis: (a) symmetrical half of an equiangular spiral and (b) symmetrical half of a polar surface.

Fig. 1(b). The segment BGE represents the half of the tube surface with chord BE as the axis of symmetry. The surface, thus described, may be termed as a 'polar surface', for convenience.

Condensation phenomenon in such a situation can be effectively modeled by invoking fundamental conservation equations for mass, momentum and the condensable species, with suitable interfacial boundary conditions. However, certain simplifying assumptions are made, by appealing to the physical nature of the problem being addressed, as follows:

- 1. The condensed liquid film formed on the tube surface is thin enough so that temperature distribution within the same can be taken as linear.
- 2. The thin condensed film moves with a very low velocity such that non-linear inertia terms in the momentum conservation equation can be neglected, i.e., the film Reynolds number is very low.
- 3. The liquid phase flow is incompressible and laminar.
- 4. Thermodynamic equilibrium prevails at the liquid– vapour interface.
- 5. Vapour phase is otherwise stagnant except having only radial flow velocity due to mass diffusion.
- 6. The wall surface temperature  $T_w$  is constant.
- 7. Film surface and wall surface have the same radius of curvature  $R_c(\theta)$  at all angular locations.

For further description, we adopt of the following coordinate system notations: ' $x_1$ ' coordinate for a direction tangential to the tube surface, and ' $y_1$ ' coordinate corresponding to a radial direction measured with respect to the tube surface. The governing equation for linear momentum conservation in the liquid phase, in that case, can be written as

$$
\mu \frac{d^2 v_{x_1}}{dy_1^2} = -(\rho - \rho_v) g \cos \left(\frac{\pi}{2} - \tan^{-1} m - \theta\right) + \frac{dp_\sigma}{dx_1} \tag{1}
$$

where  $dp_{\sigma}/dx_1$  is the pressure gradient due to surface tension. The term  $p_{\sigma}$  can be described from a balance of surface tension and pressure forces at the interface as

$$
p_{\sigma} = \frac{\sigma}{R_{\rm c}}\tag{2}
$$

where  $\sigma$  is the surface tension coefficient and  $R_c$  is the local radius of curvature. In polar coordinates,  $R_c$  can be described as

$$
R_{\rm c} = \frac{\left\{ R_{\rm p}^2 + \left(\frac{dR_{\rm p}}{d\theta}\right)^2 \right\}^{3/2}}{\left\{ R_{\rm p}^2 + 2\left(\frac{dR_{\rm p}}{d\theta}\right)^2 - R_{\rm p} \frac{d^2R_{\rm p}}{d\theta^2} \right\}}
$$
(3a)

The above can be utilized to calculate the surface tension originated pressure gradient, by noting that

$$
\frac{dp_{\sigma}}{dx_1} = -\left(\frac{\sigma}{R_c^2}\right) \left(\frac{dR_c}{d\theta}\right) \left(\frac{d\theta}{dx_1}\right)
$$
(3b)

and

$$
\frac{\mathrm{d}x_1}{\mathrm{d}\theta} = \sqrt{R_p^2 + \left(\frac{\mathrm{d}R_p}{\mathrm{d}\theta}\right)^2} \tag{3c}
$$

where  $R_p = ae^{m\theta}$ . With the help of above relationships, the liquid phase momentum conservation equation can be solved, in consistency with the following boundary conditions:

$$
v_{x_1} = 0
$$
 at  $y_1 = 0$  and  $\frac{dv_{x_1}}{dy_1} = 0$  at  $y_1 = \delta$ 

where  $\delta$  is the condensate film thickness. This leads to the following velocity profile within the liquid film:

$$
v_{x_1} = \frac{(\rho - \rho_v)g}{\mu} y_1 \left( \delta - \frac{y_1}{2} \right)
$$
  
 
$$
\times \left[ \cos \left( \frac{\pi}{2} + \tan^{-1} m + \theta \right) + \frac{\sigma m e^{-2m\theta}}{(\rho - \rho_v)ga^2(1 + m^2)} \right]
$$
(4)

<span id="page-3-0"></span>Next, we describe the continuity equation for the vapour phase, as

$$
\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} = 0
$$
\n(5)

which leads to the condition  $v_r \propto 1/r$ . Now, assuming that  $v_r = v_s(\theta)$  at  $r = ae^{m\theta} + \delta$  (liquid–vapour interface), and noting that  $\delta \ll a$ , one can write,

$$
v_r = v_s(\theta) a e^{m\theta} / r \tag{6}
$$

Using this, one can describe the energy conservation equation in the vapour phase, as follows:

$$
v_{\rm s}(\theta) \frac{a e^{m\theta}}{r} \frac{\partial T_{\rm v}}{\partial r} = \alpha_{\rm v} \left( \frac{\partial^2 T_{\rm v}}{\partial r^2} + \frac{1}{r} \frac{\partial T_{\rm v}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T_{\rm v}}{\partial \theta^2} \right) \tag{7}
$$

where the subscript 'v' refers to the vapour phase. Analogously, the species conservation equation takes a form:

$$
v_s(\theta) \frac{a e^{m\theta}}{r} \frac{\partial C}{\partial r} = D \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{1}{r^2} \frac{\partial^2 C}{\partial \theta^2} \right)
$$
(8)

where  $C$  is the mass fraction of the condensable. The boundary conditions for the solution of Eqs. (7) and (8) are as follows:

at 
$$
r = ae^{m\theta}
$$
,  $T_v = T_{si}$  and  $C = C_{si}$   
at  $r = r_{\infty}$ ,  $T = T_{\infty}$  and  $C = C_{\infty}$ 

The value of  $r_{\infty}$  for the present computation has been taken as  $r_{\infty} = 10a e^{m\theta}$  which equals to the ten times of tube radius at any angular location. A change in its value beyond this has a negligible influence (less than 2%) in the field variables. It needs to be noted here that interfacial values of temperature and concentration are thermodynamically linked through phase equilibrium conditions. This can be written in two steps, by first expressing the vapour pressure  $(p_v)$  as a function of interfacial saturation temperature  $(T_{si})$  and then, expressing the condensable mass fraction at interface  $(C_{si})$  in terms of the vapour pressure. Here, we execute these two steps by utilizing pertinent closeformed expressions [\[14\],](#page-6-0) as

$$
\frac{p_{\rm v}}{p_{\rm total}} = \exp\left[\frac{h_{\rm fg,a}}{R} \left\{ \frac{T_{\rm si} - T_{\rm sa}}{T_{\rm si} T_{\rm sa}} \right\} - \frac{0.38}{T_{\rm c}} \ln \frac{T_{\rm si}}{T_{\rm sa}} - \frac{0.118}{T_{\rm c}^2} \left\{ T_{\rm si} - T_{\rm sa} \right\} \right]
$$
(9)

where  $h_{fg,a}$  is the latent heat of condensation of steam at a reference temperature  $T_{sa}$ ,  $T_c$  is the critical temperature of water, and  $p_{total}$  is the total pressure of the mixture. The corresponding interfacial mass fraction of condensables can then be calculated as [\[14\]:](#page-6-0)

$$
C_{\rm si} = \frac{1}{1 + \frac{M_{\rm air}}{M_{\rm water}} \left(\frac{p_{\rm total}}{p_{\rm v}} - 1\right)}\tag{10}
$$

The radial velocity at liquid–vapour interface,  $v_s$ , can be expressed from consideration of impermeability of the interface to non-condensable species, as

$$
v_{\rm s} = \frac{-D \frac{\partial C}{\partial r}|_{r=a e^{m\theta} + \delta}}{1 - C_{\rm si}} \tag{11}
$$

Eq. (11) originates from a physical consideration that the non-condensable gas is transported from the bulk to the interface by a convective flow, which carries the condensable water vapour as well. Since the interface is impermeable to the non-condensable gas, it must be removed at the same rate at which it arrives at the interface, in order to maintain steady-sate conditions. This removal is accomplished by a diffusive flow back to the bulk, leading to an interfacial concentration gradient.

Finally, with all the above information, we invoke the mass conservation principle, applied to a differential control volume within the liquid film, as

$$
\dot{m}_{x_1+dx_1} = \dot{m}_{x_1} + \dot{m}_c dx_1 \tag{12}
$$

where  $\dot{m}_c$  is the rate of condensation. The above can be simplified by invoking Taylor series expansions to obtain:

$$
\frac{\mathrm{d}}{\mathrm{d}x_1} \left[ \int_0^\delta \rho v_{x_1} \, \mathrm{d}y_1 \right] = \dot{m}_c \tag{13}
$$

Again,  $\dot{m}_c$  can be described as

$$
\dot{m}_{\rm c} = k \frac{T_{\rm si} - T_{\rm w}}{\delta h_{\rm fg}} - \frac{k_{\rm v}}{h_{\rm fg}} \frac{\partial T_{\rm v}}{\partial r}\bigg|_{\rm interface} \tag{14}
$$

where  $h_{fg}$  is a function of local interfacial temperature, and  $T_w$  is the tube surface temperature. Here we use the following variation of  $h_{\text{fg}}$ :

$$
h_{\rm fg} = h_{\rm fg,a} \left[ \frac{T_{\rm c} - T_{\rm si}}{T_{\rm c} - T_{\rm sa}} \right]
$$
 (15)

Eq. (13) can be simplified with the aid of Eqs. [\(4\) and \(14\)](#page-2-0) to yield an ordinary differential equation of the form:

$$
\frac{\mathrm{d}\chi}{\mathrm{d}\theta} = \frac{A}{\chi^{1/3}} \left[ f(\theta) \right]^{1/3} + B \tag{16}
$$

where

$$
A = \frac{-3\mu ak(T_{si} - T_w)}{\rho(\rho - \rho_v)gh_{fg}}
$$
(16a)

$$
B = \frac{3\mu ak_{\rm v} \frac{\partial T}{\partial r}|_{r=ae^{m\theta}+\delta}}{\rho(\rho-\rho_{\rm v})gh_{\rm fg}}
$$
(16b)

$$
f(\theta) = \cos\left(\frac{\pi}{2} + \tan^{-1}m + \theta\right) + \frac{\sigma m e^{-2m\theta}}{(\rho - \rho_v)ga^2(1 + m^2)}
$$
(16c)

and

$$
\chi = \delta^3 f(\theta) \tag{16d}
$$

Eq. (16) can be numerically integrated to obtain the growth of film thickness. To determine the initial value of  $\lambda$  at  $\theta = \tan^{-1m}$  we have to know the film thickness  $\delta$  at  $\theta = \tan^{-1m}$  (the upper most point of the tube surface). Since the condensate mass flow rate is zero at this location, the initial film thickness is taken to be the same as that for

<span id="page-4-0"></span>the condensation of a pure vapour over the same polar tube surface. Therefore, we have, following Dutta et al. [\[9\]](#page-6-0),

$$
\delta_{\text{at }\theta=\tan^{-1}m}=\delta_0=\left[\frac{3M(1+m^2)^{\frac{1}{2}}e^{m\theta}}{1-\frac{2\sigma m^2e^{-2m\tan^{-1}m}}{(\rho-\rho_v)ga^2(1+m^2)}}\right]^{1/4}
$$

Hence

$$
\lambda_{\text{at } \theta = \tan^{-1} m} = \lambda_0 = \delta_0^3 f(\tan^{-1m})
$$
 (16e)

It has also been observed that any value of  $\lambda_0$  close to the value given by Eq. (16e) makes a negligible influence in the solution of  $\delta$  as a function of  $\theta$ . It can be noted here that the values of interfacial temperature and concentration are not known a priori. These values are obtained by an iterative updation of the field variables, through numerical solution of Eqs. [\(7\) and \(8\)](#page-3-0). This is done by assuming first a logical guess value of  $C_{si}$  which may correspond to a value less than  $C_{\infty}$  and the corresponding value of  $T_{\rm si}$  satisfying Eqs. [\(9\) and \(10\).](#page-3-0) Then the values of  $C_{si}$  and  $T_{si}$  are continuously updated through repeated numerical solutions of Eqs. [\(7\) and \(8\),](#page-3-0) satisfying simultaneously Eqs. [\(14\) and](#page-3-0) [\(16\)](#page-3-0) and an additional equation  $\dot{m}_c = \rho_g V_s$  for the use of Eq. [\(16\)](#page-3-0) to update the value of  $T_{si}$ . Thus the converged values of  $C_{si}$ ,  $T_{si}$  and field variables are finally obtained.

The variation of film thickness, as determined as Eq. [\(16\)](#page-3-0) can be subsequently utilized to estimate the heat transfer coefficients, and hence, the Nusselt number. The overall heat transfer coefficient  $(h)$ , by convention, is defined by the following:

$$
q'' = \frac{k(T_{\rm si} - T_{\rm w})}{\delta} = h(T_{\infty} - T_{\rm w})\tag{17}
$$

The condensation heat transfer coefficient  $(h_c)$  can be described as

$$
h_{\rm c} = \frac{\dot{m}_{\rm c} h_{\rm fg}}{(T_{\infty} - T_{\rm si})} \tag{18}
$$

The overall heat transfer coefficient  $(h)$  is also related to the film heat transfer coefficient  $(h_f)$  and gas phase heat transfer coefficient  $(h_{\mathcal{G}})$  and the condensation heat transfer coefficient  $(h_c)$  by the following:

$$
\frac{1}{h} = \frac{1}{h_{\rm f}} + \frac{1}{h_{\rm c} + h_{\rm g}}\tag{19}
$$

where

$$
h_{\rm f} = \frac{k(T_{\rm si} - T_{\rm w})}{\delta(T_{\rm si} - T_{\rm w})} = \frac{k}{\delta} \tag{19a}
$$

$$
h_{\rm g} = \frac{|-k_{\rm v}(\frac{\partial T_{\rm v}}{\partial r})|}{(T_{\infty} - T_{\rm si})}
$$
(19b)

The Nusselt number, Nu, is defined on the basis of overall heat transfer coefficient, h, as

$$
Nu = \frac{h(2R)}{k} \tag{20}
$$

where  $R$  is the radius of an equivalent circular tube which has the same surface area as that of the polar surface under consideration. It can be written, following Dutta et al. [\[9\]](#page-6-0) that

$$
R = \frac{a(1+m^2)^{1/2}}{\pi m} \left( e^{(\pi/2+\phi)m} - e^{m\tan^{-1}m} \right)
$$
 (21)

where the angle  $\phi$  is obtained from the following equation [\[9\]](#page-6-0):

$$
\cos \phi = \frac{a e^{m \tan^{-1} m} \sin \left(\tan^{-1} m\right)}{a e^{m \left(\pi/2 + \phi\right)}}
$$
\n(22)

The average Nusselt number can be calculated as

$$
Nu_{\rm av} = \frac{\int_{\tan^{-1}m}^{\phi} Nua\mathrm{e}^{m\theta} \,\mathrm{d}\theta}{\int_{\tan^{-1}m}^{\phi} a\mathrm{e}^{m\theta} \,\mathrm{d}\theta} \tag{23}
$$

## 3. Results and discussion

It has been established earlier by Dutta et al. [\[9\]](#page-6-0) that the pertinent dimensionless parameters influencing the Nusselt number in condensation of pure vapour over a polar tube surface are  $(Ra/Ja)^{1/4}$ , m and  $N<sub>\sigma</sub>$ . For the purpose of continuity and completeness, Fig. 2 is drawn after Dutta et al. [\[9\]](#page-6-0) to show the variations of local Nusselt number for a polar surface with  $m = 1$  and that for a circular tube surface  $(m = 0)$  whose surface area is same as that of a polar tube surface with  $m = 1$ . These results are in excellent accordance with the predictions reported in Dutta et al. [\[9\]](#page-6-0).

[Fig. 3](#page-5-0) depicts the variations of local Nusselt number along the tube surface for different mass fractions of noncondensable species in the bulk mixture at free stream (expressed in percentage) conditions. It is observed that the presence of even a marginal amount of non-condensable species reduces the Nusselt number considerably. The variations in overall heat transfer coefficient (h) based average Nusselt number (Nu) and condensation heat transfer



Fig. 2. Variation of local Nusselt number with  $x_1/L$ , for a condensing system that does not contain any non-condensable.

<span id="page-5-0"></span>

Fig. 3. Variation of local Nusselt number with  $x_1/L$ , corresponding to different percentage of non-condensables, with  $m = 1$ .



Fig. 4. Variation of average Nusselt number, with mass fraction of noncondensables, for  $m = 1$ .

coefficient  $(h_c)$  based average Nusselt number  $(Nu_c)$  with concentration of non-condensable species are shown in Fig. 4. There is a drastic reduction in both the Nusselt numbers with an increase in amount of non-condensable species. The value of  $Nu_c$  is always higher than that of  $Nu$ . This is because of the fact that the condensation heat transfer coefficient is defined on the basis of a temperature difference of  $(T_{\infty} - T_{\text{si}})$  (refer to Eq. [\(18\)\)](#page-3-0), which is much lower than the temperature difference of  $(T_{\infty} - T_{w})$  on the basis of which the overall heat transfer coefficient,  $h$ , is defined (refer to Eq. [\(17\)\)](#page-4-0). This reduction in heat transfer coefficient can be attributed to the fact that the condensable vapour has to diffuse through a body of vapour–gas mixture to the interface, because of which, there occurs a considerable drop in vapour concentration and tempera-



Fig. 5. Variation of temperature and mass fraction of the condensable within the gaseous (vapour  $+$  non-condensable) phase, in a direction normal to the interface, at  $\theta = \pi/2$ .

ture from their free stream values to the corresponding interfacial values, as shown in Fig. 5. This results in a reduction in the rate of condensation and finally in the overall heat transfer coefficient.

Fig. 6 depicts the most important feature of the present work. An enhancement in heat transfer coefficient for a polar tube surface with respect to a circular tube, due to aiding effective gravity forces and surface tension driven favourable pressure gradients, has already been established in the literature [\[9\]](#page-6-0), where it has also been recognized that the heat transfer coefficient increases with an increase in value of  $m$ , with other significant parameters ( $Ra$ ,  $Ja$  and  $N<sub>\sigma</sub>$ ) remaining the same. What has been missing in the literature, however, is any systematic study exploring a possible utilization of this heat transfer augmentation



Fig. 6. Variation of ratio of average Nusselt number with and without condensables, as a function of mass fraction of non-condensables, for different values of m.

<span id="page-6-0"></span>capability to arrest a drastic reduction in the condensation heat transfer rate that otherwise would occur in presence of non-condensables. [Fig. 6](#page-5-0) shows the variations in the ratio of average Nusselt number with non-condensable species  $(Nu_{av, with})$  to that without non-condensable species  $(Nu_{av, without})$ , against mass fraction of non-condensable species. It is observed that the above ratio increases with an increase in value of  $m$ , for the same mass fraction of non-condensable species. This implies physically that for a polar tube surface, with an increase in  $m$ , the enhancement in heat transfer coefficient in presence of noncondensable species is more than the corresponding proportional enhancement in the same in absence any non-condensable species. This additional gain suggests an advantage in use of polar tube surfaces with higher values of m, more specifically for condensation of a vapour in presence of non-condensable gases, obviously within the constraints imposed by tube manufacturing processes.

#### 4. Conclusions

From the present study, following major conclusions can be drawn:

- Heat transfer coefficient in film condensation in presence of non-condensable species over non-circular tube surfaces with progressively increasing radius of curvature in the direction of gravity has been theoretically investigated. A polar surface comprising a segment of an equiangular spiral curve, generated symmetrically on a vertical chord, has been considered.
- The average Nusselt number for a polar surface is always higher than that corresponding to a circular one of the same surface area, due to an effective gravity force component and favourable surface tension driven pressure gradients acting on the liquid film. On the other hand, the presence of a marginal amount of noncondensable species reduces drastically the average Nusselt number from its value corresponding to a pure vapour (devoid of any non-condensable species).
- An increase in value of  $m$  (the parameter defining the equiangular spiral curve) increases the average Nusselt number,  $Nu_{av}$ , for a polar surface. With an increase in m, the enhancement in  $Nu_{av}$  for a polar surface over that

for a circular surface, in case of condensation with the presence of non-condensable species, is more pronounced than the corresponding proportional enhancement in  $Nu_{av}$  without the presence of any noncondensable species.

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